**Homework 11**

**P21.1.4** Determine the LT of the following functions: (a) (*t −* 1)*u*(*t −* 2); (b) (*t −* 2)*u*(*t −* 1). Verify the results by expressing each function as the sum of a delayed ramp function and a delayed step function and deriving their LTs.



**Solution:** L

L.

(a) L

L. The function is equivalent to



(*t* – 2)*u*(*t* – 2) + *u*(*t* – 2), as shown.

(b) L

L . The function is equivalent to

(*t* – 1)*u*(*t* – 1) – *u*(*t* – 1), as shown.

**P21.1.7** Determine the LT of  given that the LT of is .

**Solution:** The function whose LT is required is *t* times the given function. Hence, from the multiplication by *t* property, the LT is , where . Thus, .

**P21.1.8** Using the division by *t* property and referring to a table of integrals, show that

L



**Solution:** L



**P21.2.4** Determine the LT of *f*(*t*) in Figure P21.2.4.

**Solution:** The derivative of the function is a shown, where *f*(1)(*t*) = *u*(*t*) – 0.5*u*(*t –* 1) – 0.5*u*(*t* – 3) – 2*δ*(t – 1); *sF*(*s*) = 

.



**P21.2.8** Determine the LT of the derivative of *f*(*t*) in Figure P21.2.8.

**Solution.** *Method 1*: From direct integration,  =   . From the differentiation property, .

*Method 2*: in terms of ramp and step functions, *f*(*t*) = *tu*(*t*) – (*t* – 1)*u*(*t* – 1) + *u*(*t* – 1) – *u*(*t* – 2) – (*t* – 2)*u*(*t* – 2) + (*t* – 3)*u*(*t* – 3); , as before.

*Method 3*: The derivative of the function is as shown. It is expressed as: -*δ*(*t*) + *u*(*t*) – *u*(*t* – 1) + *δ*(*t –* 1) – *δ*(*t –* 2) – *u*(*t* – 2) + *u*(*t* – 3). *F*(1)(*s*) = , as before.



**P21.2.10** Figure P21.2.10 shows two identical consecutive pulses each of duration *a*, the second pulse being inverted with respect to the first. If *F*(*s*) is the LT of the two pulses shown, determine, in terms of *F*(*s*), the LT of *f*(*t*) shifted to the left by *a*.



**Solution:** Let *G*(*s*) be the LT of a single pulse that extends from *t* = 0 to *t* = *a*. Then, , or . When shifted to the left, the LT of *f*(*t* + *a*) is 

**P21.3.5** If *f*(*t*) = (cos*πt*/2)*u*(*t*) and *g*(*t*) is an infinite train of unit impulses of period 1, as shown in Figure P21.3.5, determine the LT of the



product *f*(*t*)×*g*(*t*).

**Solution:** The product is zero at odd values of *t*, at which cos*πt*/2 = 0. The product is: *δ*(*t*) + *δ*(*t* – 4) + *δ*(*t* – 8), etc. at *t* = 0, 4s, 8s, 12s, etc., and is -*δ*(*t* – 2) - *δ*(*t* – 6) - *δ*(*t* – 10), etc. at *t* = 2s, 6s, 10s, etc. Both trains are of period 4 s. Their LTs are, respectively,  and . The sum is .

**P21.3.6** Using the time-shift property, show that the Laplace transform of a single half sinusoid described by and elsewhere, is: Verify the result by direct integration using partial integration. Deduce that the Laplace transform of a full-wave rectified waveform of amplitude can be expressed as:



**Solution:** *f*(*t*) = *A*msin*ωtu*(*t*) + *Am*sin*ωu*; hence, *F*(*s*) =  . By direct integration: *F*(*s*) =  =  , so *F*(*s*) = . According to Equation 21.2.25, the LT of a full-wave rectified waveform is =

